

# Supplementary Material

## A Model Predictive Control Approach for Reach Redirection in Virtual Reality

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### 1 Model Validation Data

The development and evaluation of a Minimum Jerk-based redirection model is presented in Section 3 of our paper. Here we provide a more thorough visual comparison of the model simulation results against experimental data. We present the mean hand trajectories for all 5 horizontal redirection levels in the validation dataset (0, 4, 8, 12, 16 cm), averaged across all participants.

#### 0 cm Redirection: Average Real & Simulated Hand Trajectories

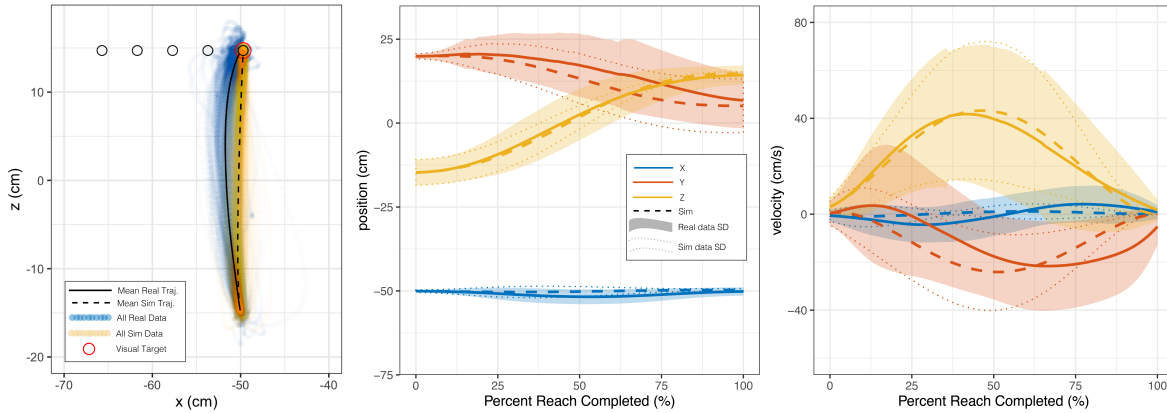


Figure 1: (Left) Mean real and simulated reach paths for 0 cm redirection trials, averaged across all participants and overlaid over data from all experimental and simulation trials. (Middle) Mean and standard deviations for real and simulated hand position trajectories. (Right) Mean and standard deviations for real and simulated hand velocity profiles.

## 4 cm Redirection: Average Real & Simulated Hand Trajectories

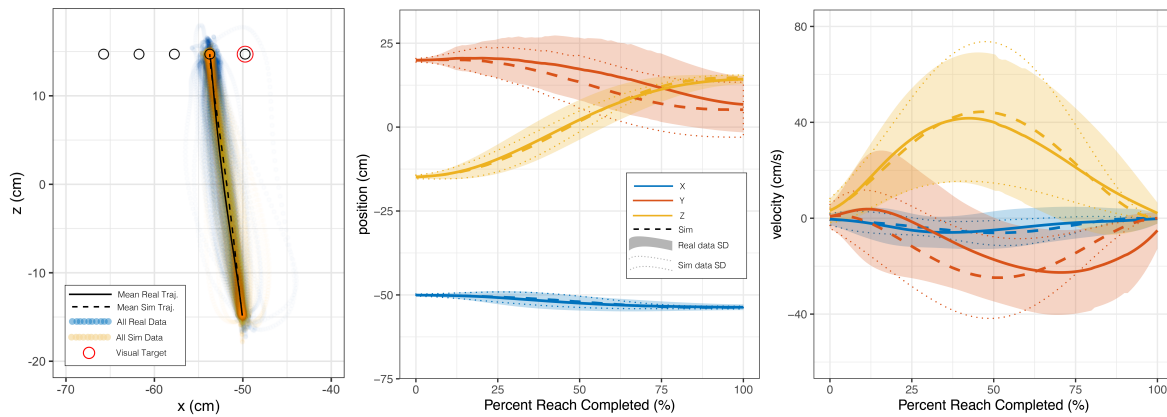


Figure 2: (Left) Mean real and simulated reach paths for 4 cm redirection trials, averaged across all participants and overlaid over data from all experimental and simulation trials. (Middle) Mean and standard deviations for real and simulated hand position trajectories. (Right) Mean and standard deviations for real and simulated hand velocity profiles.

## 8 cm Redirection: Average Real & Simulated Hand Trajectories

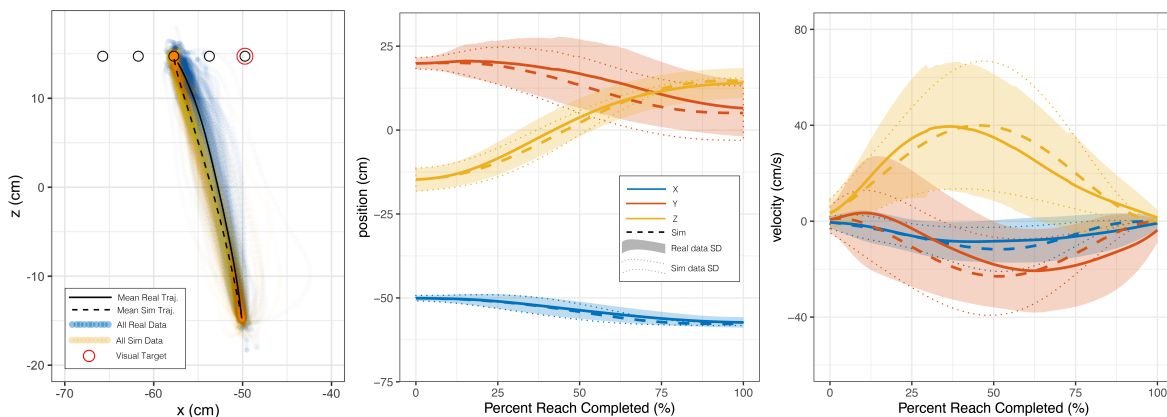


Figure 3: (Left) Mean real and simulated reach paths for 8 cm redirection trials, averaged across all participants and overlaid over data from all experimental and simulation trials. (Middle) Mean and standard deviations for real and simulated hand position trajectories. (Right) Mean and standard deviations for real and simulated hand velocity profiles.

## 12 cm Redirection: Average Real & Simulated Hand Trajectories

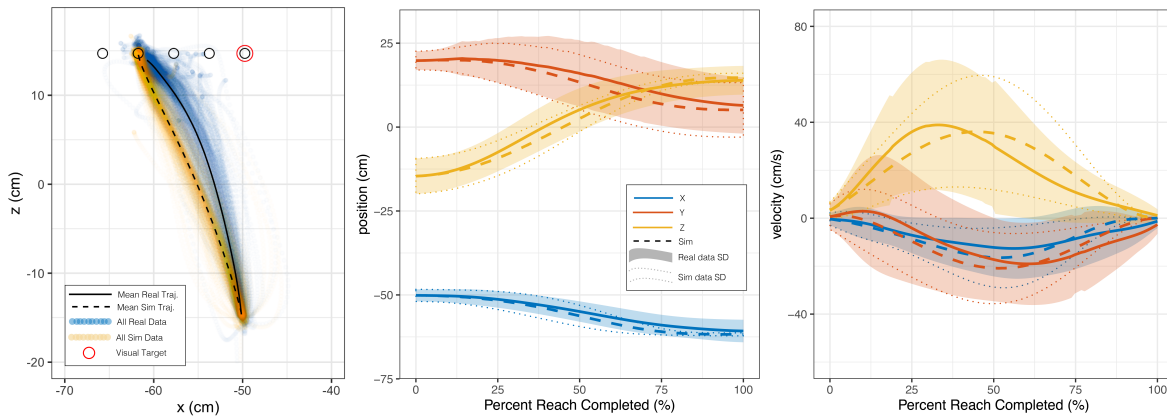


Figure 4: (Left) Mean real and simulated reach paths for 12 cm redirection trials, averaged across all participants and overlaid over data from all experimental and simulation trials. (Middle) Mean and standard deviations for real and simulated hand position trajectories. (Right) Mean and standard deviations for real and simulated hand velocity profiles.

## 16 cm Redirection: Average Real & Simulated Hand Trajectories

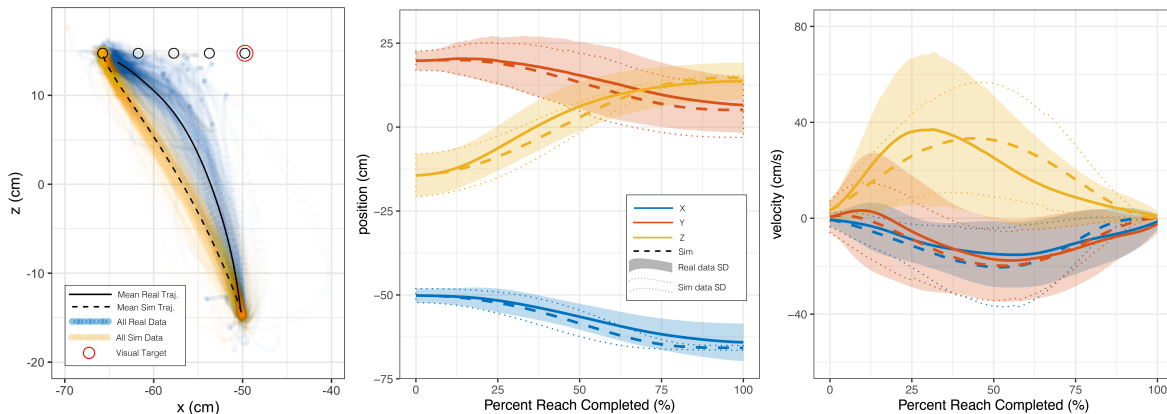


Figure 5: (Left) Mean real and simulated reach paths for 16 cm redirection trials, averaged across all participants and overlaid over data from all experimental and simulation trials. (Middle) Mean and standard deviations for real and simulated hand position trajectories. (Right) Mean and standard deviations for real and simulated hand velocity profiles.

## 2 Statistical Contrasts

In this section, we provide the tabulated results of all statistical contrasts completed during the analysis of our study results (Section 7 of our paper), including: endpoint error, reach time, root-mean-squared path error (RMSE), dimensionless-squared jerk (DSJ) of hand trajectory, noticeability of warping (perception of real and virtual hand movements matching), and tolerability of warping.

Table 1: Endpoint Error Contrasts

Study	Level	<i>df</i>	<i>t</i>	<i>p</i>
Endpoint	0 - 4	241	1.89	0.359
Endpoint	0 - 8	241	2.33	0.125
Endpoint	0 - 16	241	-0.52	1.000
Endpoint	4 - 8	241	0.44	1.000
Endpoint	4 - 16	241	-2.42	0.099
Endpoint	4 - 16	241	-2.85	<b>0.028</b>
Path	MPC-P <sub>0.1</sub> - MPC-P <sub>0.05</sub>	365	1.94	0.161
Path	MPC-P <sub>0.1</sub> - TPS	365	3.41	<b>0.002</b>
Path	MPC-P <sub>0.05</sub> - TPS	365	1.48	0.424
Path	0 - 0.2	365	-0.93	1.000
Path	0 - 0.4	365	-2.08	0.229
Path	0 - 0.6	365	-5.83	< <b>.0001</b>
Path	0.2 - 0.4	365	-1.15	1.000
Path	0.2 - 0.6	365	-4.90	< <b>.0001</b>
Path	0.4 - 0.6	365	-3.75	<b>0.001</b>

Table 2: Reach Time Contrasts

Study	Level	<i>df</i>	<i>t</i>	<i>p</i>
Endpoint	0 - 4	244	1.90	0.349
Endpoint	0 - 8	244	0.96	1.000
Endpoint	0 - 16	244	-1.04	1.000
Endpoint	4 - 8	244	-0.95	1.000
Endpoint	4 - 16	244	-2.94	<b>0.022</b>
Endpoint	4 - 16	244	-1.99	0.284
Path	MPC-P <sub>0.1</sub> - MPC-P <sub>0.05</sub>	365	-3.48	<b>0.002</b>
Path	MPC-P <sub>0.1</sub> - TPS	365	3.34	<b>0.003</b>
Path	MPC-P <sub>0.05</sub> - TPS	365	6.83	< <b>.0001</b>
Path	0 - 0.2	365	-1.12	1.000
Path	0 - 0.4	365	-2.15	0.195
Path	0 - 0.6	365	-4.89	< <b>.0001</b>
Path	0.2 - 0.4	365	-1.03	1.000
Path	0.2 - 0.6	365	-3.77	<b>0.001</b>
Path	0.4 - 0.6	365	-2.74	<b>0.039</b>

Table 3: RMSE Contrasts

Study	Level	<i>df</i>	<i>t</i>	<i>p</i>
Path	MPC-P <sub>0.1</sub> - MPC-P <sub>0.05</sub>	365	0.00	1.000
Path	MPC-P <sub>0.1</sub> - TPS	365	2.95	<b>0.010</b>
Path	MPC-P <sub>0.05</sub> - TPS	365	2.95	<b>0.010</b>
Path	0 - 0.2	365	-1.03	1.000
Path	0 - 0.4	365	-1.77	0.467
Path	0 - 0.6	365	-2.78	<b>0.034</b>
Path	0.2 - 0.4	365	-0.75	1.000
Path	0.2 - 0.6	365	-1.76	0.476
Path	0.4 - 0.6	365	-1.01	1.000

Table 4: DSJ Contrasts

Study	Level	<i>df</i>	<i>t</i>	<i>p</i>
Endpoint	0°: MPC-E - HR	239	1.32	0.190
Endpoint	4°: MPC-E - HR	239	1.28	0.202
Endpoint	8°: MPC-E - HR	239	-2.55	<b>0.012</b>
Endpoint	16°: MPC-E - HR	239	-0.79	0.433
Path	MPC-P <sub>0.1</sub> - MPC-P <sub>0.05</sub>	364	-2.53	<b>0.035</b>
Path	MPC-P <sub>0.1</sub> - TPS	364	1.18	0.713
Path	MPC-P <sub>0.05</sub> - TPS	364	3.71	<b>0.001</b>
Path	0 - 0.2	364	0.87	1.000
Path	0 - 0.4	364	0.55	1.000
Path	0 - 0.6	364	-3.26	<b>0.007</b>
Path	0.2 - 0.4	364	-0.32	1.000
Path	0.2 - 0.6	364	-4.13	<b>0.0003</b>
Path	0.4 - 0.6	364	-3.81	<b>0.001</b>

Table 5: Trial Hand Match Contrasts

Study	Level	<i>df</i>	<i>t</i>	<i>p</i>
Endpoint	0 - 4	241	-1.82	0.421
Endpoint	0 - 8	241	-2.53	0.073
Endpoint	0 - 16	241	7.17	<. <b>0.0001</b>
Endpoint	4 - 8	241	-0.71	1.000
Endpoint	4 - 16	241	8.99	<. <b>0.0001</b>
Endpoint	4 - 16	241	9.70	<. <b>0.0001</b>
Path	0: MPC-P <sub>0.1</sub> - MPC-P <sub>0.05</sub>	365	3.36	<b>0.003</b>
Path	0: MPC-P <sub>0.1</sub> - TPS	365	-2.52	<b>0.036</b>
Path	0: MPC-P <sub>0.05</sub> - TPS	365	-5.88	<. <b>0.0001</b>
Path	0.2: MPC-P <sub>0.1</sub> - MPC-P <sub>0.05</sub>	365	3.92	<b>0.0003</b>
Path	0.2: MPC-P <sub>0.1</sub> - TPS	365	-3.36	<b>0.003</b>
Path	0.2: MPC-P <sub>0.05</sub> - TPS	365	-7.28	<. <b>0.0001</b>
Path	0.4: MPC-P <sub>0.1</sub> - MPC-P <sub>0.05</sub>	365	2.80	<b>0.016</b>
Path	0.4: MPC-P <sub>0.1</sub> - TPS	365	-1.26	0.625
Path	0.4: MPC-P <sub>0.05</sub> - TPS	365	-4.06	<b>0.0002</b>
Path	0.6: MPC-P <sub>0.1</sub> - MPC-P <sub>0.05</sub>	365	2.10	0.109
Path	0.6: MPC-P <sub>0.1</sub> - TPS	365	0.56	1.000
Path	0.6: MPC-P <sub>0.05</sub> - TPS	365	-1.54	0.373

Table 6: Block Hand Match Contrasts

Study	Level	<i>df</i>	<i>t</i>	<i>p</i>
Path	MPC-P <sub>0.1</sub> - MPC-P <sub>0.05</sub>	14	2.69	<b>0.053</b>
Path	MPC-P <sub>0.1</sub> - TPS	14	0.00	1.000
Path	MPC-P <sub>0.05</sub> - TPS	14	-2.69	<b>0.053</b>

Table 7: Block Tolerability Contrasts

Study	Level	<i>df</i>	<i>t</i>	<i>p</i>
Path	MPC-P <sub>0.1</sub> - MPC-P <sub>0.05</sub>	14	1.68	0.345
Path	MPC-P <sub>0.1</sub> - TPS	14	-1.68	0.345
Path	MPC-P <sub>0.05</sub> - TPS	14	-3.36	<b>0.014</b>